The λ-calculus: typed and untyped

https://cl-teaching.uibk.ac.at/26w/lc

Assessed Exercises — Lecture 6

Due on Friday 27 November 2026 at 11:00 (Innsbruck time).
Submit your solutions through the OLAT website

Exercise 1 (2 points)

Find *a* derivation in NI with conclusion $f : M \vdash \lambda a.f(aa) : C$, for some *concrete* multi type M and linear type C.

Exercise 2 (4 points)

Find *all* derivations in NI with conclusion $f : M \vdash \lambda a.f(aa) : C$, for *any* multi type M and linear type C.

Exercise 3 (4 points)

Write an untyped λ-term t such that, when applied to a list of Church numerals, it returns their product.

Verify that $t\,[\underline{1}, \underline{2}, \underline{3}] \to_\beta^* \underline{6}$ by writing the β-reduction sequence step-by-step, possibly writing $\underline{\mathit{mult}}\,\underline{m}\,\underline{n} \to_\beta^* \underline{m \cdot n}$ as a single step (for any $m, n \in \mathbb{N}$, where $\underline{\mathit{mult}}$ is a term representing multiplication on natural numbers).

Hint. Use the encoding for lists seen in Lecture 5. Get inspired by non-assessed Exercises 7–9 from Lecture 5.