The λ-calculus: typed and untyped

https://cl-teaching.uibk.ac.at/26w/lc

Assessed Exercises — Lecture 5

Due on Friday 20 November 2026 at 11:00 (Innsbruck time).
Submit your solutions through the OLAT website

Exercise 1 (2 points)

Explain why confluence of β-reduction in STLC (in Curry-style or Church-style) *a priori* does not follow immediately from confluence of β-reduction on the untyped λ-calculus. Prove that indeed confluence of β-reduction in STLC actually follows from confluence of β-reduction on the untyped λ-calculus, using another property of STLC too.

Exercise 2 (4 points)

1. (2 points) Prove that Church-Rosser holds in STLC (in Curry-style or Church-style), that is, if s and t are typable in STLC and $s =_\beta t$ then there is a term u typable in STLC such that $s \to_\beta^* u \,{}_\beta^*\!\leftarrow t$.
2. (2 points) Deduce that if $\Gamma \vdash_{\text{Church}} s : A$ and $\Gamma \vdash_{\text{Church}} t : B$ and $s =_\beta t$ then $A = B$. Disprove this property in $\mathsf{STLC}_{\text{Curry}}$.

Exercise 3 (4 points)

In the untyped λ-calculus, prove that there is a closed term t (that is, $\mathsf{fv}(t) = \emptyset$) such that $t \to_\beta^* \lambda x.xtxtx$.

Hint. Use the fixpoint combinator Θ seen in Lecture 4. Get inspired by the way we represented the factorial in the untyped λ-calculus seen in Lecture 5.