The λ-calculus: typed and untyped

https://cl-teaching.uibk.ac.at/26w/lc

Assessed Exercises — Lecture 4

Due on Friday 13 November 2026 at 11:00 (Innsbruck time).
Submit your solutions through the OLAT website

Exercise 1 (3 points)

Prove rigorously that $\Rightarrow_\beta$ is closed under substitution, that is, if $s \Rightarrow_\beta s'$ and $t \Rightarrow_\beta t'$ then $s\{t/x\} \Rightarrow_\beta s'\{t'/x\}$.

Exercise 2 (3 points)

Show that $\to_\beta$ is not closed under substitution, i.e. find some terms t, t', s such that $t \to_\beta t'$ and $s\{t/x\} \not\to_\beta s\{t'/x\}$.

Exercise 3 (4 points)

Let $(A, \to)$ be an ARS such that, for all $t, s_1, s_2 \in A$, if $s_1 \leftarrow t \to s_2$ then $s_1 \to r \leftarrow s_2$ for some $r \in A$. Prove that $\to$ is confluent.

Hint. To have a tool to proceed by induction, use the notion of n-step reduction $\to^n$ (for any $n \in \mathbb{N}$) defined by: $t \to^n s$ if there is $(r_i)_{0 \le i \le n}$ such that $t = r_0$, $s = r_n$ and $r_i \to r_{i+1}$ for all $0 \le i < n$.