The λ-calculus: typed and untyped

https://cl-teaching.uibk.ac.at/26w/lc

Assessed Exercises — Lecture 3

Due on Sunday 8 November 2026 at 23:00 (Innsbruck time).
Submit your solutions through the OLAT website

Exercise 1 (2 points)

Given the λ-term $t = \lambda f^{X \Rightarrow X}.\lambda x^{X}.f(fx)$ in Church-style, find two *distinct* derivations $\mathcal{D} \rhd_{\mathsf{STLC}_{\mathrm{Church}}} \Gamma \vdash t : A$ and $\mathcal{D}' \rhd_{\mathsf{STLC}_{\mathrm{Church}}} \Gamma' \vdash t : A'$ for some environments Γ, Γ' and some types A, A'.

Do they contradict the proposition about the uniqueness of type and derivation in $\mathsf{STLC}_{\mathrm{Church}}$ seen in Lecture 3, p. 19? Justify briefly your answer.

Hint. The derivations $\mathcal{D}$ and $\mathcal{D}'$ must be distinct, but possibly $\Gamma = \Gamma'$ and/or $A = A'$.

Exercise 2 (4 points)

In the saturation lemma for $\mathsf{STLC}_{\mathrm{Curry}}$ seen in Lecture 3, p. 28, prove Point 2 when the type B is X (a ground type).

Hint. The solution to a very similar non-assessed exercise from Lecture 3 may be useful (if used, its proof should be part of the submission).

Exercise 3 (4 points)

In the saturation lemma for $\mathsf{STLC}_{\mathrm{Church}}$ seen in Lecture 3, p. 28, prove Point 3 when the type B is X (a ground type).

Hint. The solution to a very similar non-assessed exercise from Lecture 3 may be useful (if used, its proof should be part of the submission).