

The λ -calculus: typed and untyped

<https://cl-teaching.uibk.ac.at/26w/1c>

Assessed Exercises — Lecture 1

Due on Friday 23 October 2026 at 11:00 (Innsbruck time).
Submit your solutions through the [OLAT website](#)

Exercise 1 (3 points)

Prove that $(X \Rightarrow Y) \Rightarrow X \vdash (X \Rightarrow Y) \Rightarrow Y$ by using both ND and ND_{seq}.

Exercise 2 (2 points)

Let us extend minimal logic to conjunction. To this end, consider the language defined by the BNF grammar below

$$A, B, C ::= X \mid A \Rightarrow B \mid A \wedge B$$

where $\wedge$ stands for the conjunction (“and”). Consider the system ND ^{$\wedge$} obtained by adding the inference rules below to ND.

$$\frac{A \quad B}{A \wedge B} \wedge_i$$

$$\frac{A \wedge B}{A} \wedge_{e_1}$$

$$\frac{A \wedge B}{B} \wedge_{e_2}$$

Define the corresponding inference rules for the system ND_{seq} ^{$\wedge$} , that is, define the version of the inference rules above that is compatible with the ND_{seq} formalism.

Exercise 3 (5 points)

Define the notions of $\wedge$ -cut-formula and $\wedge$ -redex in the system ND ^{$\wedge$} concerning the rules $\wedge_i$, $\wedge_{e_1}$ and $\wedge_{e_2}$ from Exercise 2. Define the cut-elimination steps for a $\wedge$ -redex. State and prove the cut elimination theorem for ND ^{$\wedge$} .

Hint. Get inspired from the corresponding notions, theorems and proofs for ND seen in Lecture 1.