universität innsbruck

Computability Theory	26W	LVA 703317
Lecture 6		November 30, 2026

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. Solutions for bonus exercises must be submitted separately. The (strict) deadline is 10 am on November 30.

Exercises

⟨2⟩ 1. Prove the lemma on slide 11.

⟨2⟩ 2. Prove that the set of natural numbers which are not powers of two is diophantine.

⟨1⟩ 3. Prove that diophantine sets are closed under union and intersection.

⟨1⟩ 4. Construct a primitive recursive function $f(x, y)$ such that $\varphi_{f(x,y)} = \varphi_x \circ \varphi_y$ for all $x, y \geqslant 0$.

⟨1⟩ 5. Define a *universal* partial recursive function $\varphi\colon \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ which generates all partial recursive functions of any arity, in the sense that for every n-ary partial recursive function f there exists a natural number e such that $f(x_1, \ldots, x_n) \simeq \varphi(e, \langle x_1, \ldots, x_n \rangle)$.

⟨2⟩ 6. Construct a unary partial recursive function which cannot be extended to a total recursive function.

Bonus Exercise

⟨3⟩ 7. Prove the second theorem on slide 31.