universität innsbruck

Computability Theory	26W	LVA 703317
Lecture 5		November 23, 2026

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. The (strict) deadline is 10 am on November 23.

Exercises

⟨2⟩ 1. Complete the proof of Kleene's normal form theorem.

⟨1⟩ 2. Define a primitive recursive function arity such that $\mathsf{arity}(x) = n$ if x is the index of an n-ary partial recursive function and 0 otherwise.

⟨2⟩ 3. Let us write $(\nu\, i)\,(g(i, \vec{y}) = 0)$ for the smallest i such that $g(i, \vec{y}) = 0$ and, for all $j < i$, either $g(j, \vec{y})\uparrow$ or $g(j, \vec{y}) > 0$.

   (a) Construct a partial recursive function φ such that $(\mu\, i)\,(\varphi(i, \vec{y}) = 0)$ and $(\nu\, i)\,(\varphi(i, \vec{y}) = 0)$ are different functions.

   (b) Show that the class of partial recursive functions is not closed under the ν operator.

⟨2⟩ 4. Prove that there is no primitive recursive predicate *recursive* that tests whether its argument is the index of a total recursive function.

⟨2⟩ 5. Prove that for every two recursive functions $f, g\colon \mathbb{N}^2 \to \mathbb{N}$ there exist $a, b \in \mathbb{N}$ such that $\varphi_{f(a,b)} \simeq \varphi_a$ and $\varphi_{g(a,b)} \simeq \varphi_b$.