

Solved exercises must be marked and solutions (as a single PDF file) uploaded in [OLAT](#). Solutions for bonus exercises must be submitted separately. The (strict) deadline is 10 am on October 19.

Exercises

- ⟨2⟩ 1. Let c be an arbitrary natural number.
 - (a) Prove that $\text{ack}(c, x)$ is a primitive recursive function.
 - (b) Is $\text{ack}(x, c)$ primitive recursive?
- ⟨2⟩ 2. Complete the proof on [slide 19](#) by showing $f(x, \vec{y}) < \text{ack}(c', x + \max\{\vec{y}\})$ for all $x, \vec{y} \in \mathbb{N}$.
- ⟨2⟩ 3. Compute indices for multiplication and exponentiation.
- ⟨1⟩ 4. Which primitive recursive function has index $\langle 4, \langle 1 \rangle, \langle 3, \langle 1 \rangle, \langle 2, 3, 1 \rangle \rangle \rangle$?
- ⟨2⟩ 5. Which number is the encoding of the computation of $\text{ack}(2, 1) = 5$?

Bonus Exercise

- ⟨3⟩ 6. Consider the primitive recursive function $\beta: \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$ defined as

$$\beta(a, i) = \pi_1(a) \bmod (1 + (i + 1)\pi_2(a))$$

Prove that for every finite sequence $a_0, \dots, a_n$ of natural numbers there exists a natural number a such that $\beta(a, i) = a_i$ for all $0 \leq i \leq n$.