

Solved exercises must be marked and solutions (as a single PDF file) uploaded in [OLAT](#). The (strict) deadline is 10 am on October 12.

Exercises

- ⟨3⟩ 1. Prove that the following functions are primitive recursive:
- (a) $f(x) = \begin{cases} x \div 2 & \text{if } x \text{ is even} \\ 3x + 1 & \text{if } x \text{ is odd} \end{cases}$
 - (b) $|x - y|$
 - (c) The function $g(x)$ that returns the natural number y such that $y \leq \sqrt{x} < y + 1$.
 - (d) $\text{lcm}(x, y)$
 - (e) The function $\Pi(x)$ that returns the number of primes less than or equal to x .
- ⟨2⟩ 2. Consider the function $\pi'(x, y) = ((x + y)^2 + 3x + y) \div 2$.
- (a) Prove that π' is a bijection between $\mathbb{N}^2$ and $\mathbb{N}$.
 - (b) Define corresponding primitive recursive extraction functions π'_1 and π'_2 .
- ⟨2⟩ 3. Let c be an arbitrary natural number.
- (a) Let $f: \mathbb{N}^{n+1} \rightarrow \mathbb{N}$ be a primitive recursive function with $n > 0$. Prove that the function $g: \mathbb{N}^n \rightarrow \mathbb{N}$ defined by $g(x_1, \dots, x_n) = f(x_1, \dots, x_n, c)$ is primitive recursive.
 - (b) Let $h: \mathbb{N}^2 \rightarrow \mathbb{N}$ be primitive recursive. Prove that the function $f: \mathbb{N} \rightarrow \mathbb{N}$ defined by the recursive equations
$$\begin{aligned} f(0) &= c \\ f(x + 1) &= h(f(x), x) \end{aligned}$$
is primitive recursive.
- ⟨2⟩ 4. Define a primitive recursive function $\text{sort}: \mathbb{N} \rightarrow \mathbb{N}$ such that $\text{sort}(\langle x_1, \dots, x_n \rangle) = \langle x_{\sigma(1)}, \dots, x_{\sigma(n)} \rangle$ for a permutation σ of $1, \dots, n$ with the property that $x_{\sigma(1)} \leq \dots \leq x_{\sigma(n)}$.