



Automata and Logic

Aart Middeldorp and Johannes Niederhauser

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 - ① Automata and Logic
 - ② Constraint Solving
 - ③ Cryptography
 - ④ High-Performance Computing
 - ⑤ Optimisation and Numerical Computation
 - ⑥ Signal Processing and Algorithmic Geometry

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 - ⑥ Signal Processing and Algorithmic Geometry
- ▶ other master modules with theory content (**Logic and Learning** specialization):
 - ▶ Program and Resource Analysis (WM 8)
 - ▶ Computability Theory (WM 9)
 - ▶ Advanced Functional Programming (WM 20)
 - ▶ **The Lambda-Calculus, Typed and Untyped** (WM 20)

Outline

1. Introduction

Organisation

AI Policy

Contents

2. Basic Definitions

3. Deterministic Finite Automata

4. Intermezzo

5. Closure Properties

6. Further Reading

 with session ID **3017 2066** for anonymous questions



Important Information

- ▶ LVA 703302 (VO 2) + 703303 (PS 2)

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Time and Place

VO	Monday	8:30–10:00	HSB 9	(AM)
PS	Friday	8:30–10:00	SR 12	(JN)

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Consultation Hours

Aart Middeldorp	3M07	Monday	11:30–13:00
Johannes Niederhauser	3M03	Wednesday	10:00–11:30

Schedule

week 1	05.10 & 09.10	week 7	30.11 & 04.12
week 2	12.10 & 23.10	week 8	07.12 & 11.12
week 3	19.10 & 30.10 & 06.11	week 9	14.12 & 18.12 & 08.01
week 4	09.11 & 13.11	week 10	11.01 & 15.01
week 5	16.11 & 20.11	week 11	18.01 & 22.01
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Grading — Vorlesung

- ▶ first exam on January 25

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Grading — Vorlesung

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- ▶ de-registration is possible until 10:00 on January 22
- ▶ second exam on February 22; third exam on May 21 (on demand)

$$\text{score} = \min\left(\frac{2}{3}(E + P) + B, 100\right)$$

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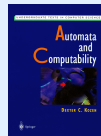
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grade : $[0, 50) \rightarrow \mathbf{5}$ $[50, 63) \rightarrow \mathbf{4}$ $[63, 75) \rightarrow \mathbf{3}$ $[75, 88) \rightarrow \mathbf{2}$ $[88, 100] \rightarrow \mathbf{1}$

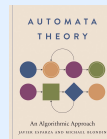
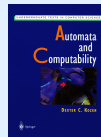
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evaluation 25W

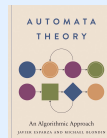
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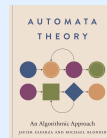
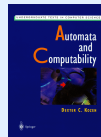


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Online Material

- ▶ solutions to selected exercises are available after they have been discussed in PS

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AI Policy — Principles

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AI use is permitted, but must be declared and comply with other restrictions (specific details to be provided by course teacher)



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- ▶ (deterministic, nondeterministic, alternating) finite automata
- ▶ regular expressions
- ▶ (alternating) Büchi automata

Logic

- ▶ (weak) monadic second-order logic
- ▶ Presburger arithmetic
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Examples

strings over $\Sigma = \{0, 1\}$: 0 0110

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languages over Σ :

- ▶ $\{\epsilon, 0, 1, 00, 01, 10, 11\}$ (all strings having at most two symbols)
- ▶ $\{x \mid x \text{ is valid program in some machine language}\}$

- ▶ **string concatenation** $x, y \in \Sigma^* \implies xy \in \Sigma^*$ is associative:
$$(xy)z = x(yz) \quad \text{for all } x, y, z \in \Sigma^*$$

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$$x^{n+1} = x^n x$$

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- ▶ $\#a(x)$ ($a \in \Sigma$, $x \in \Sigma^*$) denotes number of a 's in x

Definitions

for $A, B \subseteq \Sigma^*$

► **union**

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

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Definitions

for $A, B \subseteq \Sigma^*$

- ▶ union
- ▶ intersection
- ▶ complement

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► powers A^n ($n \in \mathbb{N}$)

$$A^0 = \{\epsilon\} \quad A^{n+1} = AA^n$$

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- ▶ powers A^n ($n \in \mathbb{N}$) $A^0 = \{\epsilon\}$ $A^{n+1} = AA^n$
- ▶ **asterate** A^* is union of all finite powers of A

$$A^* = \bigcup_{n \geq 0} A^n = \{x_1 x_2 \cdots x_n \mid n \geq 0 \text{ and } x_i \in A \text{ for all } 1 \leq i \leq n\}$$

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$$\text{▶ } A^+ = AA^* = \bigcup_{n \geq 1} A^n$$

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► $A^+ = AA^* = \bigcup_{n \geq 1} A^n$

► power set

$$2^A = \{Q \mid Q \subseteq A\}$$

Examples

- ▶ substrings of 011: 0, 1, 01, 11, 011

Examples

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- ▶ $2^{\{1, 01\}} = \{\emptyset, \{1\}, \{01\}, \{1, 01\}\}$

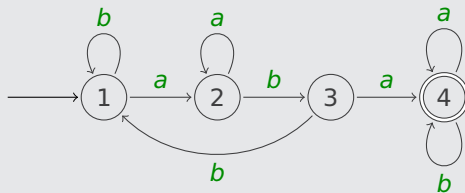
Some Useful Properties

- ▶ $\{\epsilon\}A = A\{\epsilon\} = A$
- ▶ $\emptyset A = A\emptyset = \emptyset$
- ▶ $\sim(A \cup B) = (\sim A) \cap (\sim B)$
- ▶ $\sim(A \cap B) = (\sim A) \cup (\sim B)$
- ▶ $A^{m+n} = A^m A^n$
- ▶ $A^* A^* = A^*$
- ▶ $A^{**} = A^*$
- ▶ $A^* = \{\epsilon\} \cup AA^* = \{\epsilon\} \cup A^* A$
- ▶ $\emptyset^* = \{\epsilon\}$

Outline

1. Introduction
2. Basic Definitions
- 3. Deterministic Finite Automata**
4. Intermezzo
5. Closure Properties
6. Further Reading

Example



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Definitions

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① Q : finite set of **states**

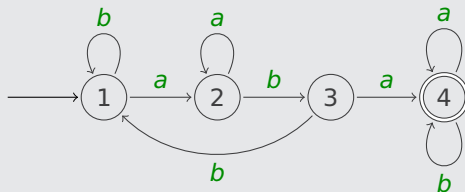
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Example

DFA $M = (Q, \Sigma, \delta, s, F)$



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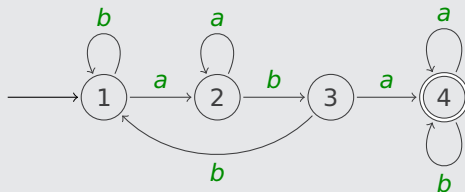
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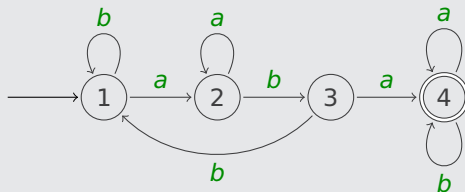
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- ⑤ $F \subseteq Q$: **final** (accept) states

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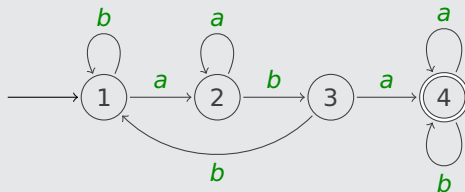
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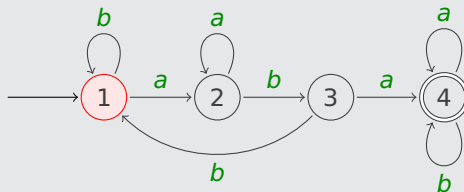
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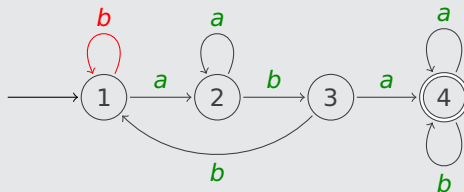
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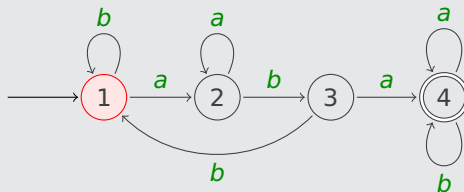
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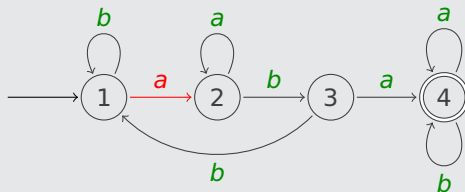
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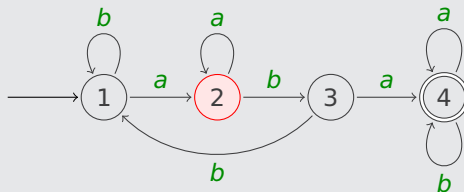
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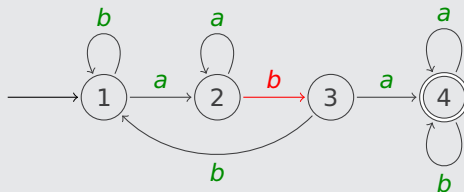
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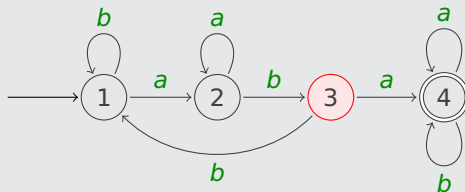
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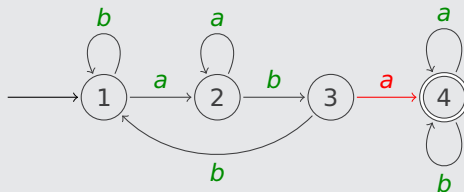
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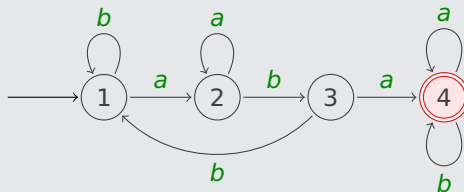
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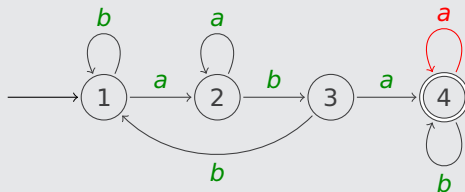
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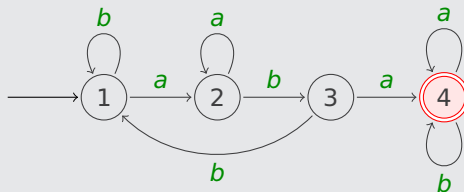
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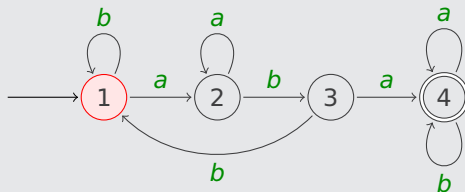
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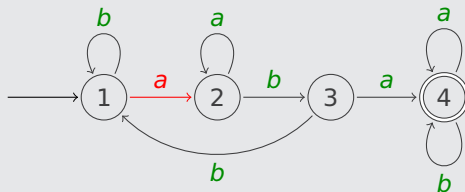
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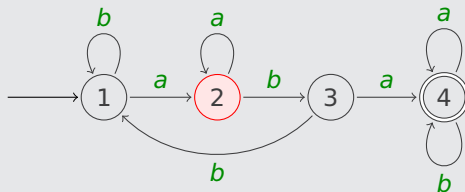
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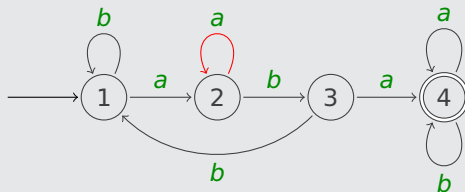
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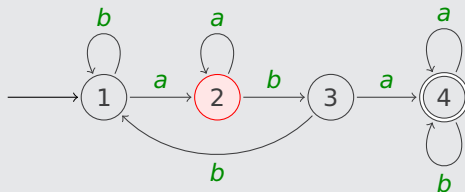
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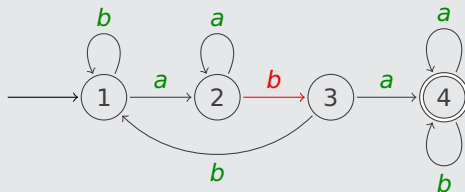
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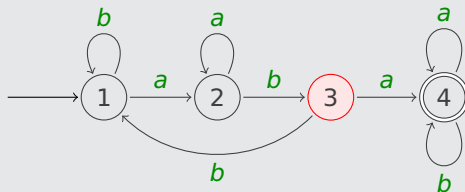
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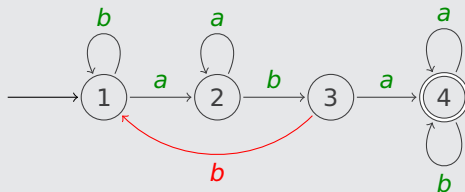
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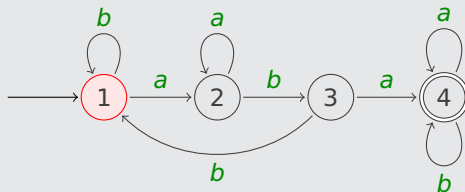
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2	2	3
3	4	1
4	4	4

$b \ a \ b \ a \ a$
 1 1 2 3 4 4

$a \ a \ b \ b \ b$
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Example

DFA $M = (Q, \Sigma, \delta, s, F)$



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2 $\Sigma = \{a, b\}$

3 $\delta: Q \times \Sigma \rightarrow Q$

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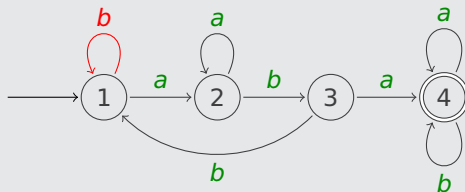
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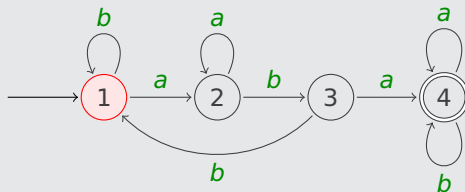
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$$\hat{\delta}(q, \epsilon) = q$$

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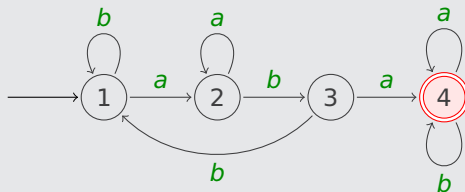
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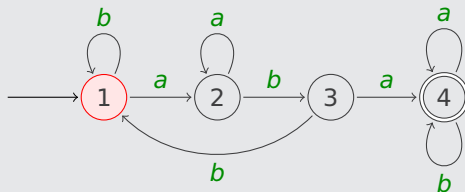
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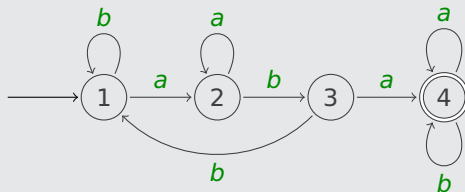
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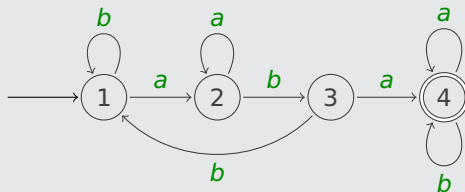
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Outline

1. Introduction
2. Basic Definitions
3. Deterministic Finite Automata
- 4. Intermezzo**
5. Closure Properties
6. Further Reading

Question

What is the language accepted by the DFA given by the following transition table ?

		<i>a</i>	<i>b</i>
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Here the arrow indicates the start state and *F* marks the final states.

- A** $\{(ab)^n \mid n \in \mathbb{N}\}$
- B** $\sim(\{a, b\}^* \{aa\})$
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- D** the set of all strings over $\{a, b\}$ with an odd number of *a*'s and an even number of *b*'s



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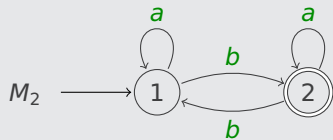
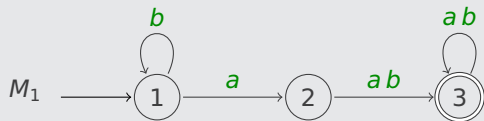
Theorem

regular sets are effectively closed under **intersection**

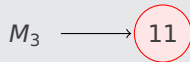
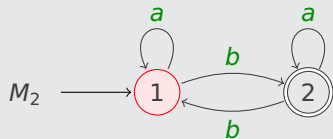
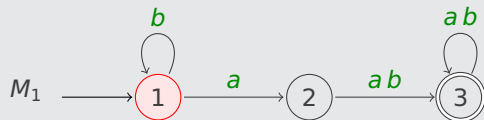
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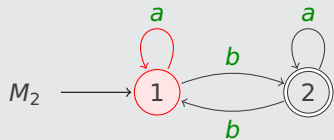
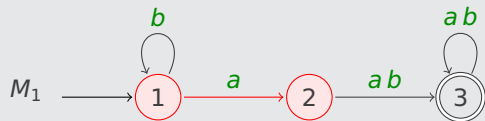
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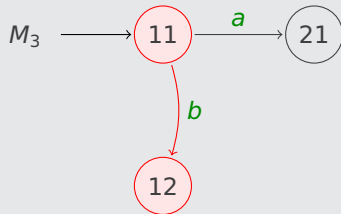
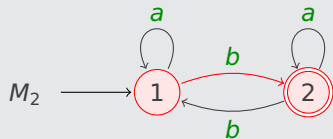
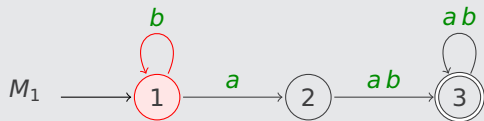
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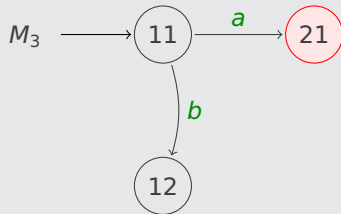
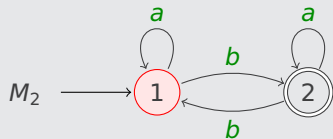
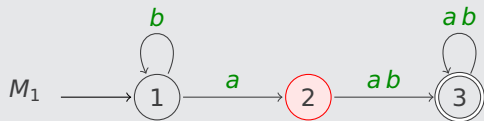
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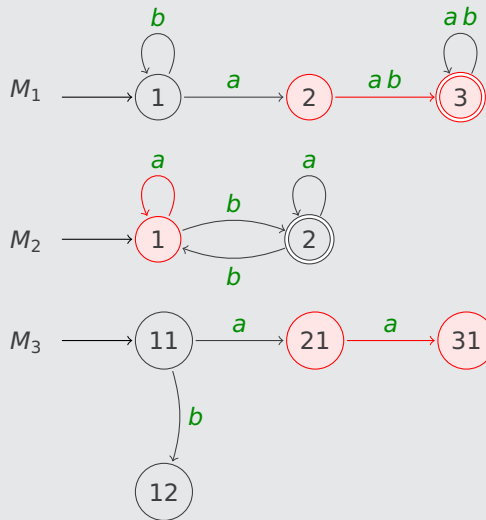
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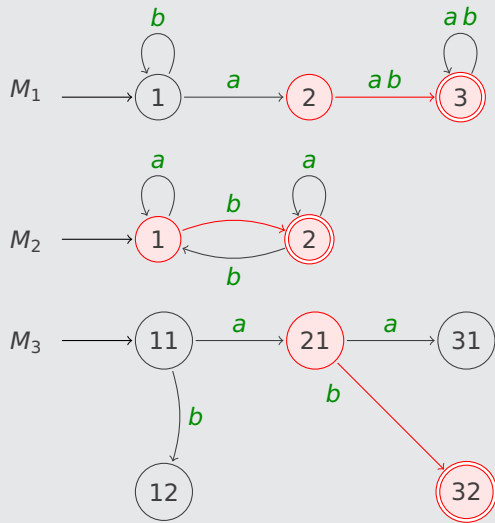
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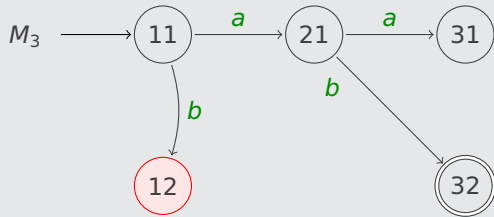
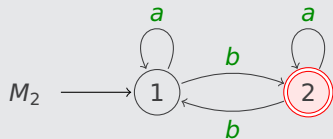
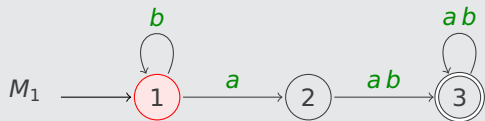
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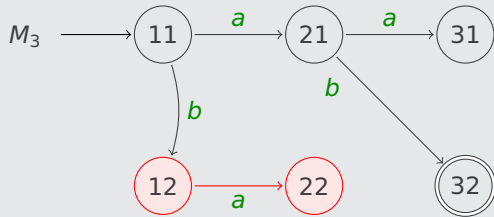
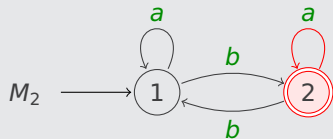
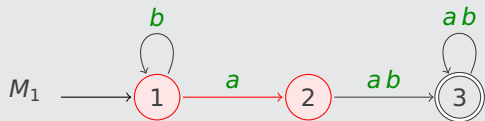
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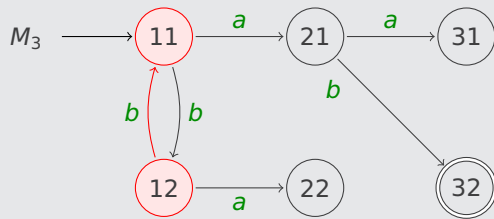
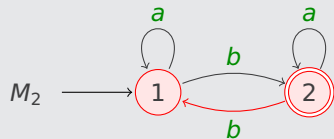
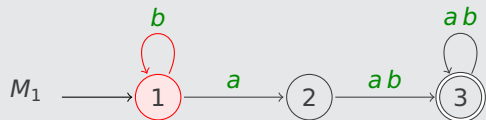
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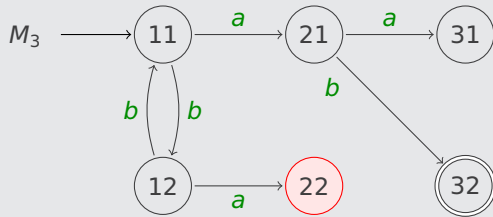
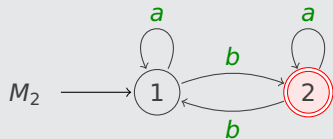
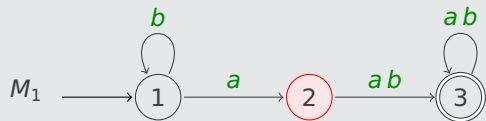
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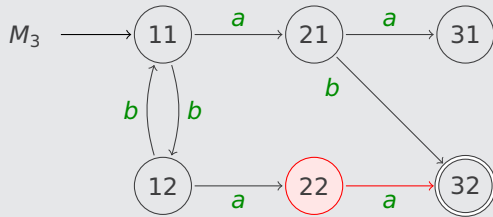
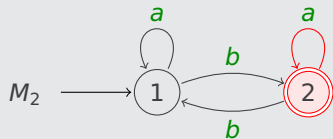
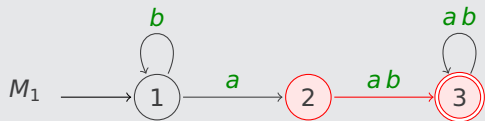
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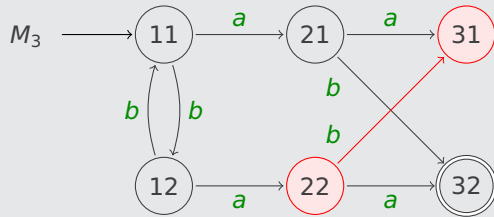
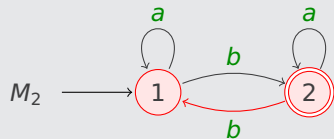
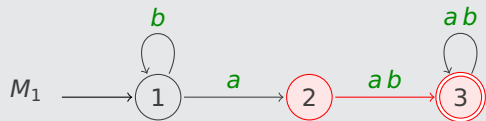
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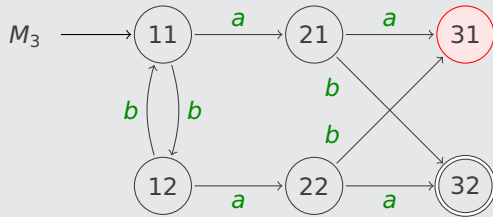
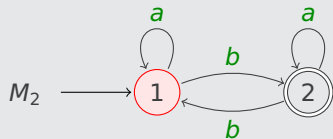
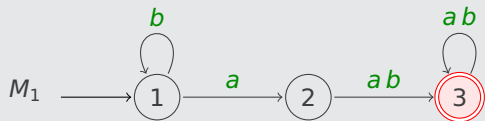
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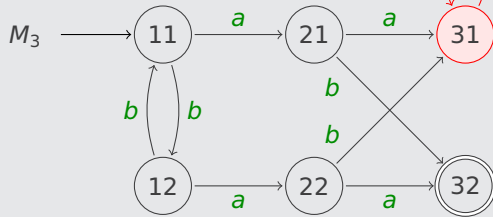
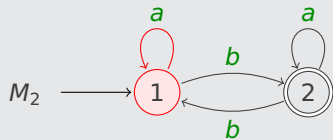
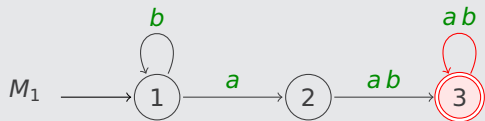
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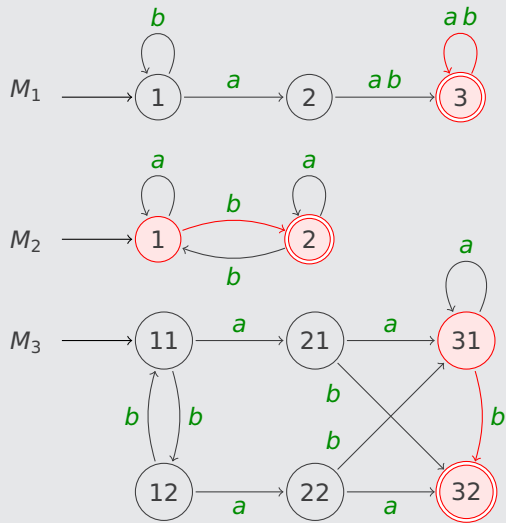
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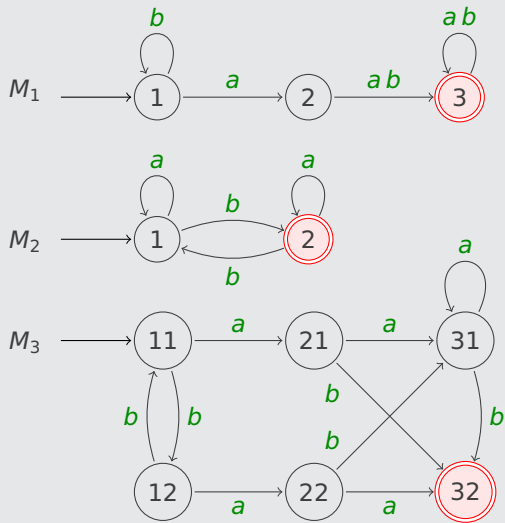
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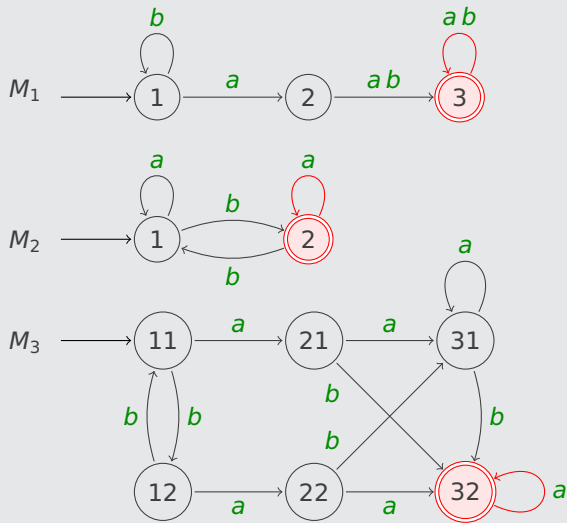
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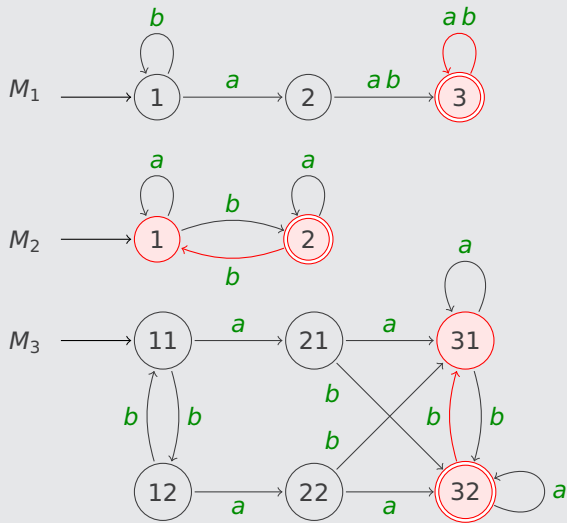
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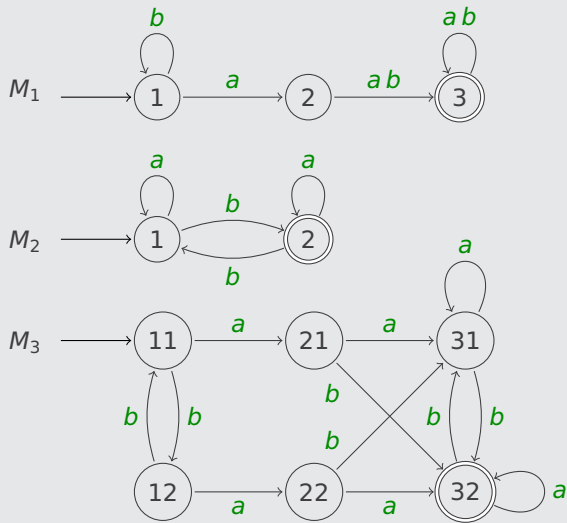
Example



Example



Example



$$L(M_3) = L(M_1) \cap L(M_2)$$

Theorem

regular sets are **effectively** closed under **intersection**

Proof

- ▶ $A = L(M_1)$ for DFA $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$
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- ▶ $A \cap B = L(M_3)$ for DFA $M_3 = (Q_3, \Sigma, \delta_3, s_3, F_3)$ with
 - ① $Q_3 = Q_1 \times Q_2 = \{(p, q) \mid p \in Q_1 \text{ and } q \in Q_2\}$
 - ② $F_3 = F_1 \times F_2$
 - ③ $s_3 = (s_1, s_2)$
 - ④ $\delta_3((p, q), a) = (\delta_1(p, a), \delta_2(q, a))$ for all $p \in Q_1, q \in Q_2, a \in \Sigma$

Theorem

regular sets are **effectively** closed under **intersection**

Proof

- ▶ $A = L(M_1)$ for DFA $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$
 $B = L(M_2)$ for DFA $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$
- ▶ $A \cap B = L(M_3)$ for DFA $M_3 = (Q_3, \Sigma, \delta_3, s_3, F_3)$ with
 - ① $Q_3 = Q_1 \times Q_2 = \{(p, q) \mid p \in Q_1 \text{ and } q \in Q_2\}$
 - ② $F_3 = F_1 \times F_2$
 - ③ $s_3 = (s_1, s_2)$
 - ④ $\delta_3((p, q), a) = (\delta_1(p, a), \delta_2(q, a))$ for all $p \in Q_1, q \in Q_2, a \in \Sigma$
- ▶ claim: $\widehat{\delta}_3((p, q), x) = (\widehat{\delta}_1(p, x), \widehat{\delta}_2(q, x))$ for all $x \in \Sigma^*$
proof of claim: easy induction on $|x|$ (on next slide)

Theorem

regular sets are **effectively** closed under **intersection**

Proof (product construction)

- ▶ $A = L(M_1)$ for DFA $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$
 $B = L(M_2)$ for DFA $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$
- ▶ $A \cap B = L(M_3)$ for DFA $M_3 = (Q_3, \Sigma, \delta_3, s_3, F_3)$ with
 - ① $Q_3 = Q_1 \times Q_2 = \{(p, q) \mid p \in Q_1 \text{ and } q \in Q_2\}$
 - ② $F_3 = F_1 \times F_2$
 - ③ $s_3 = (s_1, s_2)$
 - ④ $\delta_3((p, q), a) = (\delta_1(p, a), \delta_2(q, a))$ for all $p \in Q_1, q \in Q_2, a \in \Sigma$
- ▶ claim: $\widehat{\delta}_3((p, q), x) = (\widehat{\delta}_1(p, x), \widehat{\delta}_2(q, x))$ for all $x \in \Sigma^*$
proof of claim: easy induction on $|x|$ (on next slide)

Proof of Claim

claim: $\widehat{\delta}_3((p, q), x) = (\widehat{\delta}_1(p, x), \widehat{\delta}_2(q, x))$ for all $x \in \Sigma^*$

► base case: $|x| = 0$

Proof of Claim

claim: $\widehat{\delta}_3((p, q), x) = (\widehat{\delta}_1(p, x), \widehat{\delta}_2(q, x))$ for all $x \in \Sigma^*$

► base case: $|x| = 0$ and thus $x = \epsilon$

Proof of Claim

claim: $\widehat{\delta}_3((p, q), x) = (\widehat{\delta}_1(p, x), \widehat{\delta}_2(q, x))$ for all $x \in \Sigma^*$

► base case: $|x| = 0$ and thus $x = \epsilon$

$$\widehat{\delta}_3((p, q), x) = (p, q) = (\widehat{\delta}_1(p, x), \widehat{\delta}_2(q, x))$$

Proof of Claim

claim: $\widehat{\delta}_3((p, q), x) = (\widehat{\delta}_1(p, x), \widehat{\delta}_2(q, x))$ for all $x \in \Sigma^*$

► base case: $|x| = 0$ and thus $x = \epsilon$

$$\widehat{\delta}_3((p, q), x) = (p, q) = (\widehat{\delta}_1(p, x), \widehat{\delta}_2(q, x))$$

► induction step: $|x| > 0$

Proof of Claim

claim: $\widehat{\delta}_3((p, q), x) = (\widehat{\delta}_1(p, x), \widehat{\delta}_2(q, x))$ for all $x \in \Sigma^*$

► base case: $|x| = 0$ and thus $x = \epsilon$

$$\widehat{\delta}_3((p, q), x) = (p, q) = (\widehat{\delta}_1(p, x), \widehat{\delta}_2(q, x))$$

► induction step: $|x| > 0$ and thus $x = ya$ with $|y| = |x| - 1$

Proof of Claim

claim: $\widehat{\delta}_3((p, q), x) = (\widehat{\delta}_1(p, x), \widehat{\delta}_2(q, x))$ for all $x \in \Sigma^*$

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► induction step: $|x| > 0$ and thus $x = ya$ with $|y| = |x| - 1$

$$\widehat{\delta}_3((p, q), x) = \delta_3(\widehat{\delta}_3((p, q), y), a) \quad (\text{definition of } \widehat{\delta}_3)$$

Proof of Claim

claim: $\widehat{\delta}_3((p, q), x) = (\widehat{\delta}_1(p, x), \widehat{\delta}_2(q, x))$ for all $x \in \Sigma^*$

► base case: $|x| = 0$ and thus $x = \epsilon$

$$\widehat{\delta}_3((p, q), x) = (p, q) = (\widehat{\delta}_1(p, x), \widehat{\delta}_2(q, x))$$

► induction step: $|x| > 0$ and thus $x = ya$ with $|y| = |x| - 1$

$$\begin{aligned}\widehat{\delta}_3((p, q), x) &= \delta_3(\widehat{\delta}_3((p, q), y), a) && \text{(definition of } \widehat{\delta}_3) \\ &= \delta_3((\widehat{\delta}_1(p, y), \widehat{\delta}_2(q, y)), a) && \text{(induction hypothesis)}\end{aligned}$$

Proof of Claim

claim: $\widehat{\delta}_3((p, q), x) = (\widehat{\delta}_1(p, x), \widehat{\delta}_2(q, x))$ for all $x \in \Sigma^*$

► base case: $|x| = 0$ and thus $x = \epsilon$

$$\widehat{\delta}_3((p, q), x) = (p, q) = (\widehat{\delta}_1(p, x), \widehat{\delta}_2(q, x))$$

► induction step: $|x| > 0$ and thus $x = ya$ with $|y| = |x| - 1$

$$\begin{aligned}\widehat{\delta}_3((p, q), x) &= \delta_3(\widehat{\delta}_3((p, q), y), a) && \text{(definition of } \widehat{\delta}_3) \\ &= \delta_3((\widehat{\delta}_1(p, y), \widehat{\delta}_2(q, y)), a) && \text{(induction hypothesis)} \\ &= (\delta_1(\widehat{\delta}_1(p, y), a), \delta_2(\widehat{\delta}_2(q, y), a)) && \text{(definition of } \delta_3)\end{aligned}$$

Proof of Claim

claim: $\widehat{\delta}_3((p, q), x) = (\widehat{\delta}_1(p, x), \widehat{\delta}_2(q, x))$ for all $x \in \Sigma^*$

► base case: $|x| = 0$ and thus $x = \epsilon$

$$\widehat{\delta}_3((p, q), x) = (p, q) = (\widehat{\delta}_1(p, x), \widehat{\delta}_2(q, x))$$

► induction step: $|x| > 0$ and thus $x = ya$ with $|y| = |x| - 1$

$$\begin{aligned}\widehat{\delta}_3((p, q), x) &= \delta_3(\widehat{\delta}_3((p, q), y), a) && \text{(definition of } \widehat{\delta}_3\text{)} \\ &= \delta_3((\widehat{\delta}_1(p, y), \widehat{\delta}_2(q, y)), a) && \text{(induction hypothesis)} \\ &= (\delta_1(\widehat{\delta}_1(p, y), a), \delta_2(\widehat{\delta}_2(q, y), a)) && \text{(definition of } \delta_3\text{)} \\ &= (\widehat{\delta}_1(p, x), \widehat{\delta}_2(q, x)) && \text{(definition of } \widehat{\delta}_1 \text{ and } \widehat{\delta}_2\text{)}\end{aligned}$$

Theorem

regular sets are **effectively** closed under **complement**

Theorem

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Proof

► $A = L(M_1)$ for DFA $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$

Theorem

regular sets are **effectively** closed under **complement**

Proof

- ▶ $A = L(M_1)$ for DFA $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$
- ▶ $\sim A = \Sigma^* - A = L(M_2)$ for DFA $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$ with

Theorem

regular sets are **effectively** closed under **complement**

Proof

- ▶ $A = L(M_1)$ for DFA $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$
- ▶ $\sim A = \Sigma^* - A = L(M_2)$ for DFA $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$ with
 - ① $Q_2 = Q_1$

Theorem

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Proof

- ▶ $A = L(M_1)$ for DFA $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$
- ▶ $\sim A = \Sigma^* - A = L(M_2)$ for DFA $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$ with
 - ① $Q_2 = Q_1$
 - ② $\delta_2(q, a) = \delta_1(q, a)$ for all $q \in Q_2$ and $a \in \Sigma$

Theorem

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Proof

- ▶ $A = L(M_1)$ for DFA $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$
- ▶ $\sim A = \Sigma^* - A = L(M_2)$ for DFA $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$ with
 - ① $Q_2 = Q_1$
 - ② $\delta_2(q, a) = \delta_1(q, a)$ for all $q \in Q_2$ and $a \in \Sigma$
 - ③ $s_2 = s_1$

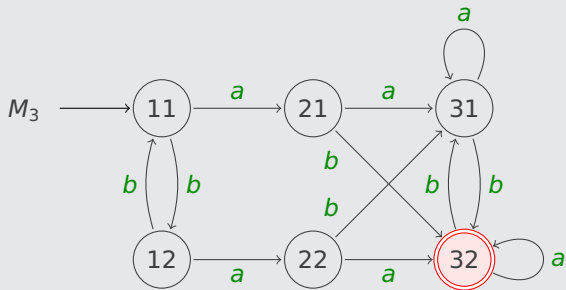
Theorem

regular sets are **effectively** closed under **complement**

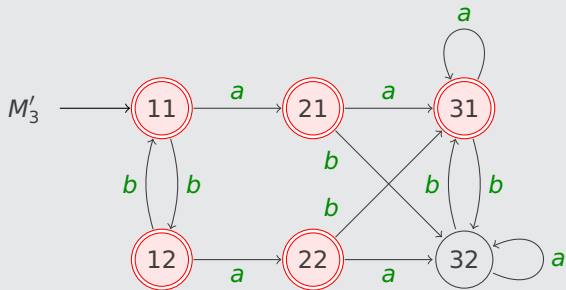
Proof

- ▶ $A = L(M_1)$ for DFA $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$
- ▶ $\sim A = \Sigma^* - A = L(M_2)$ for DFA $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$ with
 - ① $Q_2 = Q_1$
 - ② $\delta_2(q, a) = \delta_1(q, a)$ for all $q \in Q_2$ and $a \in \Sigma$
 - ③ $s_2 = s_1$
 - ④ $F_2 = Q_1 - F_1$

Example



Example



$$L(M'_3) = \sim L(M_3)$$

Theorem

regular sets are **effectively** closed under **union**

Theorem

regular sets are **effectively** closed under **union**

Proof

$$A \cup B = \sim((\sim A) \cap (\sim B))$$

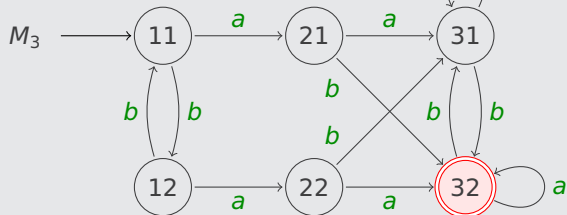
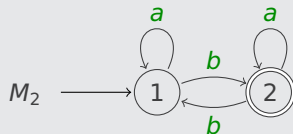
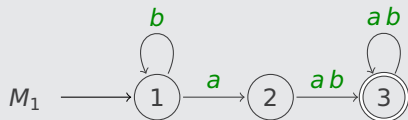
Theorem

regular sets are **effectively** closed under **union**

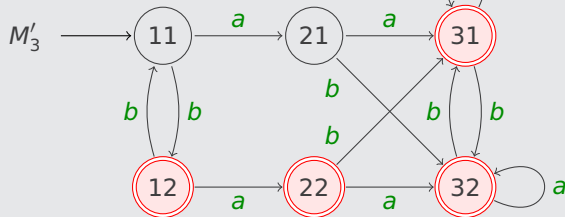
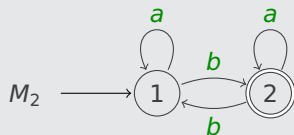
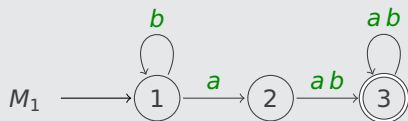
Proof (explicit construction)

- ▶ $A = L(M_1)$ for DFA $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$
 $B = L(M_2)$ for DFA $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$
- ▶ $A \cup B = L(M_3)$ for DFA $M_3 = (Q_3, \Sigma, \delta_3, s_3, F_3)$ with
 - ① $Q_3 = Q_1 \times Q_2 = \{(p, q) \mid p \in Q_1 \text{ and } q \in Q_2\}$
 - ② $F_3 = (F_1 \times Q_2) \cup (Q_1 \times F_2)$
 - ③ $s_3 = (s_1, s_2)$
 - ④ $\delta_3((p, q), a) = (\delta_1(p, a), \delta_2(q, a))$ for all $p \in Q_1, q \in Q_2, a \in \Sigma$

Example



Example



$$L(M'_3) = L(M_1) \cup L(M_2)$$

Outline

1. Introduction
2. Basic Definitions
3. Deterministic Finite Automata
4. Intermezzo
5. Closure Properties
- 6. Further Reading**

► Lectures 1–4

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Important Concepts

- ▶ alphabet
- ▶ language
- ▶ regular set
- ▶ closure properties
- ▶ product construction
- ▶ string
- ▶ DFA

- ▶ Lectures 1–4

Important Concepts

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homework for October 9

- ▶ Lectures 1–4

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homework for October 9

Solutions

... must be uploaded (PDF format) in OLAT **before 7 am on Friday**

- ▶ Lectures 1–4

Important Concepts

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- ▶ closure properties
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homework for October 9

Solutions

- ... must be uploaded (PDF format) in OLAT before 7 am on Friday
- ... bonus exercises give **bonus points**