universität innsbruck

Automata and Logic	26W	LVA 703302 + 703303
Lecture 6		November 27, 2026

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. The (strict) deadline is 7 am on November 27.

Exercises

⟨3⟩ 1. (a) Compute the minimum-state DFA for $L_a(\varphi)$ where $\varphi = (\exists\, x.\, P_a(x)) \vee (\exists\, x.\, P_b(x))$ without using MONA.

(b) Compute an automaton for $L(\varphi)$ according to the lemma on slide 21. Clearly state what the homomorphism h looks like.

⟨2⟩ 2. Read Sections 2.3, 2.4, the paragraph on "Issues in the classical approach" (page 22) and Section 3.2 of the MONA manual.[1] Run MONA on the WMSO formula $\psi = X(x) \wedge \exists\, y.\, x < y \wedge X(y)$. Turn on the automaton output and explain the result in detail. How does MONA's automaton compare to the regular expression for $L_a(\psi)$ given on slides?

⟨2⟩ 3. (a) In MONA, expressions of the form $x = y + k$ for a constant $k > 0$ are supported. Show that this does not increase the set of expressible properties in MONA by providing an equivalent WMSO formula.

(b) Use MONA to find minimal models of the following formula for $k \in \{1, 2, 3\}$:

$$P_a(0) \wedge (\forall\, x. \forall\, y.\, P_a(x) \wedge y = x + k \;\rightarrow\; P_b(y))$$

What happens when $k = 0$? Don't forget to encode that every position of a string must be either in P_a or P_b (but not both). To that end, use MONA's `allpos` declaration on a fresh second order variable which can then be used as the set of all positions.[2]

⟨3⟩ 4. Compute a minimal solution for the following puzzle with MONA:

A farmer has to take a dog, a cat, and a mouse across a river using her boat. She can only take one of the animals across at a time. If she takes the mouse, the dog will eat the cat. If she takes the dog, the cat will eat the mouse. How does she get the three animals across the river without hurting anyone?

[1] https://www.brics.dk/mona/mona14.pdf
[2] MONA manual, Section 6.5