universität innsbruck

| Automata and Logic | 26W | LVA 703302 + 703303 |
|---|---|---|
| Lecture 5 | | November 20, 2026 |

## Selected Solutions

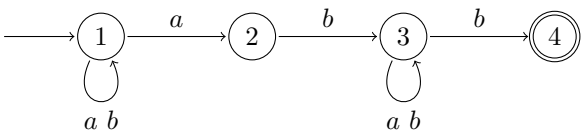
2 (a) Let $A = \{a^n b^m \mid n \neq m\}$ and consider the strings $x_i = a^i$ for $i \geqslant 0$. Let $i, j \geqslant 0$. If $i \neq j$ then $x_i b^i \notin A$ and $x_j b^i \in A$. It follows that the relation $\equiv_B$ has infinitely many equivalence classes. According to the Myhill–Nerode Theorem, $B$ cannot be regular.

Alternatively, we assume $A$ to be regular and consider $\sim A \cap L(a^*b^*) = \{a^n b^n \mid n \geqslant 0\}$. Since regular languages are closed under complement and intersection, $\{a^n b^n \mid n \geqslant 0\}$ must be regular. Hence, we have a contradiction.

(b) Let $B = \{x \in \{a, b, c\}^* \mid x = \mathbf{rev}\ x\}$ and consider the strings $x_i = a^i b$ for $i \geqslant 0$. Let $i, j \geqslant 0$. If $i \neq j$ then $x_i a^i \in B$ and $x_j a^i \notin B$. It follows that the relation $\equiv_B$ has infinitely many equivalence classes. According to the Myhill–Nerode Theorem, $B$ cannot be regular.

(c) Suppose to the contrary that PAREN is regular. Because regular sets are closed under intersection, it follows that $\text{PAREN} \cap L((^*)^*) = \{(^n)^n \mid n \geqslant 0\}$ is regular. But we already know that $\{(^n)^n \mid n \geqslant 0\}$ is not regular.

4 (a) The following NFA accepts $(a + b)^* ab (a + b)^* b$:




Using the subset construction, we obtain the equivalent DFA $M'$

| | | $a$ | $b$ |
|---|---|---|---|
| $\to$ | $A$ | $B$ | $A$ |
| | $B$ | $B$ | $C$ |
| | $C$ | $D$ | $E$ |
| | $D$ | $D$ | $E$ |
| | $EF$ | $D$ | $E$ |

where

$$A = \{1\} \qquad B = \{1, 2\} \qquad C = \{1, 3\} \qquad D = \{1, 2, 3\} \qquad E = \{1, 3, 4\}$$

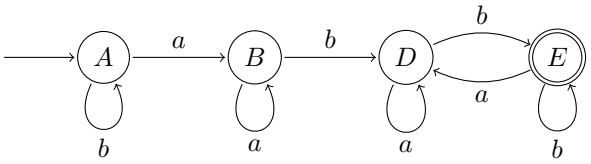
Every state in $M'$ is reachable, but $M'$ is not minimal:

| | | | | |
|---|---|---|---|---|
| $A$ | | | | |
| ✓ | $B$ | | | |
| ✓ | ✓ | $C$ | | |
| ✓ | ✓ | – | $D$ | |
| ✓ | ✓ | ✓ | ✓ | $E$ |

By collapsing states $C$ and $D$ into $D$, we obtain the following equivalent minimal DFA $M$:

| | | $a$ | $b$ |
|---|---|---|---|
| $\to$ | $A$ | $B$ | $A$ |
| | $B$ | $B$ | $D$ |
| | $D$ | $D$ | $E$ |
| | $EF$ | $D$ | $E$ |

In a picture:




(b) The equivalence classes of $\equiv_M$ correspond to the states of $M$; more precisely, each state $q$ induces the equivalence class $\{x \mid \widehat{\delta}(s,x) = q\}$. We obtain the following regular expressions:

- $(A)$ $b^*$
- $(B)$ $b^*aa^*$
- $(D)$ $b^*aa^*b(a+bb^*a)^*$
- $(E)$ $b^*aa^*ba^*b(b+aa^*b)^* = (a+b)^*ab(a+b)^*b$