universität innsbruck

Automata and Logic	26W	LVA 703302 + 703303
Lecture 4		November 13, 2026

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. The (strict) deadline is 7 am on November 13.

Exercises

⟨2⟩ 1. Consider the following DFA M:

		a	b
$\to$	$1F$	3	5
	$2F$	8	7
	3	7	2
	4	6	2
	5	1	8
	6	2	3
	7	1	4
	8	5	1

(a) Determine which states are accessible.

(b) Compute the equivalence classes of the indistinguishability relation $\approx$.

(c) Compute the minimum-state DFA for $L(M)$.

⟨2⟩ 2. Let $M_\cap$ be the DFA obtained by applying the product construction on slide 26 of lecture 1 to *minimum-state* DFAs M_1 and M_2.

(a) Show that $M_\cap$ may contain inaccessible states.

(b) Let $M'_\cap$ be obtained from $M_\cap$ by removing all inaccessible states. Is $M'_\cap$ minimal? Prove your answer.

⟨2⟩ 3. Give WMSO formulas that define the following regular sets over $\Sigma = \{a, b, c\}$:

(a) $\{abxc \mid x \in \Sigma^*\}$

(b) $\Sigma^* - L(a^*b^*c^*)$

⟨2⟩ 4. For each WMSO formula φ_i below either give a model α_i such that $\alpha_i \vDash \varphi_i$ and determine the size of α_i, or explain why φ_i is unsatisfiable:

$$\varphi_1 = \exists\, x.\, X(x) \wedge \exists\, y.\, X(y) \wedge x \neq y$$
$$\varphi_2 = (\forall\, x.\, x = 0 \to X(x)) \wedge (\forall\, x.\, X(x) \to z < x)$$
$$\varphi_3 = \varphi_1 \wedge \forall\, x.\, X(x) \to \exists\, y.\, Y(y) \wedge x < y$$

⟨2⟩ 5. Consider the alphabet $\Sigma = \{a, b\}$ and the WMSO formulas ψ_1, ψ_2 and ψ_3:

$$\psi_1 = \exists\, x.\, P_a(x) \wedge \forall\, y.\, x \neq y \to \neg P_a(y)$$
$$\psi_2 = \exists\, x.\, x = 0 \wedge \forall\, y.\, x < y \to \neg P_a(y) \wedge \neg P_b(y)$$
$$\psi_3 = \forall\, x.\, P_b(x) \to \exists\, y.\, P_a(y) \wedge x < y \wedge \neg\exists\, z.\, x < z \wedge z < y$$

For each $i \in \{1, 2, 3\}$ find a string $x_i \in \Sigma^*$ such that $x_i \in L(\psi_i)$ and $x_i \notin L(\psi_j)$ for all $1 \leqslant j \leqslant 3$ with $i \neq j$.