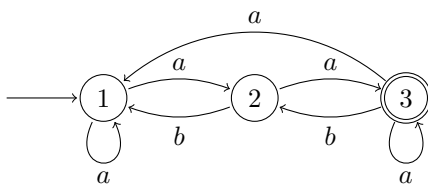


Exercises for October 30

Solved exercises must be marked and solutions (as a single PDF file) uploaded in [OLAT](#). The (strict) deadline is 7 am on October 30.

- ⟨3⟩ 1. Give regular expressions for each of the following subsets of $\{a, b, c\}^*$:
- (a) $\{x \in \{a, b\}^* \mid \#a(x) = 1\}$
 - (b) $\{x \in \{a, b\}^* \mid x \text{ does not contain the substring } ab\}$
 - (c) $\{x \mid \#a(x) \text{ is even}\}$
 - (d) $\{x \mid \#a(x) \text{ is even or } \#b(x) \text{ is odd}\}$
- ⟨2⟩ 2. Using the method on [slide 11 of lecture 3](#), transform the following automaton into an equivalent regular expression.

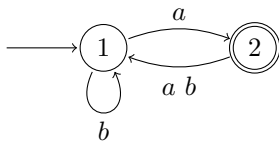


- ⟨3⟩ 3. (a) Prove that regular sets are closed under string reversal.
(b) Prove that every regular set is accepted by an NFA with exactly one final state.
- ⟨2⟩ 4. Provide concrete algorithms for the decision problems on [slide 24 of lecture 3](#). Given an estimate of the complexity of your algorithms in terms of the size (number of states, length of input string) of the instance.

Exercises for November 6

Solved exercises must be marked and solutions (as a single PDF file) uploaded in [OLAT](#). The (strict) deadline is 7 am on November 6.

- ⟨3⟩ 1. Give a regular expression for the set $\{x \in \{a, b\}^* \mid \#a(x) \text{ is even and } \#b(x) \text{ is odd}\}$.
- ⟨3⟩ 2. Let $(Q, \Sigma, \Delta, S, F)$ be an NFA. Prove the following statements:
- (a) For any $x, y \in \Sigma^*$ and $A \subseteq Q$, $\widehat{\Delta}(A, xy) = \widehat{\Delta}(\widehat{\Delta}(A, x), y)$.
 - (b) For any indexed family $(A_i)_{i \in I}$ of subsets of Q and $x \in \Sigma^*$, $\widehat{\Delta}(\bigcup_{i \in I} A_i, x) = \bigcup_{i \in I} \widehat{\Delta}(A_i, x)$.
- ⟨2⟩ 3. Prove the claims on
- (a) [slide 9 of lecture 2](#),
 - (b) [slide 18 of lecture 3](#).
- Hint:* Use the results from Exercise 2.
- ⟨2⟩ 4. Consider the DFA M



as well as the homomorphism $h: \{a, b\}^* \rightarrow \{a, b\}^*$ defined as

$$h(a) = ab$$

$$h(b) = aba$$

Construct a DFA M' such that $L(M') = \{h(x) \mid x \in L(M)\}$.