universität innsbruck

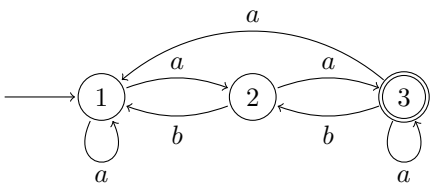
Automata and Logic	26W	LVA 703302 + 703303
Lecture 3		October 30 + November 6, 2026

Exercises for October 30

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. The (strict) deadline is 7 am on October 30.

⟨3⟩ 1. Give regular expressions for each of the following subsets of $\{a, b, c\}^*$:
   (a) $\{x \in \{a, b\}^* \mid \#a(x) = 1\}$
   (b) $\{x \in \{a, b\}^* \mid x$ does not contain the substring $ab\}$
   (c) $\{x \mid \#a(x)$ is even$\}$
   (d) $\{x \mid \#a(x)$ is even or $\#b(x)$ is odd$\}$

⟨2⟩ 2. Using the method on slide 11 of lecture 3, transform the following automaton into an equivalent regular expression.



⟨3⟩ 3. (a) Prove that regular sets are closed under string reversal.
   (b) Prove that every regular set is accepted by an NFA with exactly one final state.

⟨2⟩ 4. Provide concrete algorithms for the decision problems on slide 24 of lecture 3. Given an estimate of the complexity of your algorithms in terms of the size (number of states, length of input string) of the instance.

Exercises for November 6

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. The (strict) deadline is 7 am on November 6.

⟨3⟩ 1. Give a regular expression for the set $\{x \in \{a,b\}^* \mid \#a(x) \text{ is even and } \#b(x) \text{ is odd}\}$.

⟨3⟩ 2. Let $(Q, \Sigma, \Delta, S, F)$ be an NFA. Prove the following statements:

   (a) For any $x, y \in \Sigma^*$ and $A \subseteq Q$, $\widehat{\Delta}(A, xy) = \widehat{\Delta}(\widehat{\Delta}(A, x), y)$.

   (b) For any indexed family $(A_i)_{i \in I}$ of subsets of Q and $x \in \Sigma^*$, $\widehat{\Delta}(\bigcup_{i \in I} A_i, x) = \bigcup_{i \in I} \widehat{\Delta}(A_i, x)$.

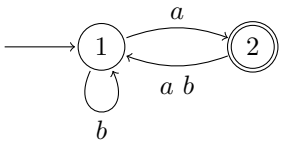
⟨2⟩ 3. Prove the claims on

   (a) slide 9 of lecture 2,

   (b) slide 18 of lecture 3.

   Hint: Use the results from Exercise 2.

⟨2⟩ 4. Consider the DFA M



   as well as the homomorphism $h\colon \{a,b\}^* \to \{a,b\}^*$ defined as

$$h(a) = ab \qquad\qquad h(b) = aba$$

   Construct a DFA M' such that $L(M') = \{h(x) \mid x \in L(M)\}$.