

Solved exercises must be marked and solutions (as a single PDF file) uploaded in [OLAT](#). The (strict) deadline is 7 am on October 23.

Exercises

- (2) 1. Construct NFAs for the following sets. Try to take advantage of nondeterminism as much as possible.
- The set of strings over $\Sigma = \{0, 1, \dots, 9\}$ such that the final digit has appeared before.
 - The set of non-empty strings over $\Sigma = \{0, 1, \dots, 9\}$ such that the final digit has *not* appeared before.

- (2) 2. Consider the following NFA $_{\epsilon}$ N :

	ϵ	a	b	c
$\rightarrow 1$	$\emptyset$	$\{1\}$	$\{2\}$	$\{1, 3\}$
2	$\{1\}$	$\{2\}$	$\{3\}$	$\emptyset$
$3F$	$\{2, 3\}$	$\{3\}$	$\emptyset$	$\{1\}$

- Compute the ϵ -closure of each state.
 - List all strings in $L(N)$ of length two or less.
 - Transform N into an equivalent DFA without inaccessible states.
- (3) 3. In this exercise we show that NFAs are exponentially more succinct than DFAs, i.e., we prove that for any $n > 0$, there exists an NFA with $n + 1$ states such that any equivalent DFA has at least 2^n states. Let $n \geq 1$ and consider the NFA $N_n = (Q, \Sigma, \Delta, S, F)$ with
- $Q = \{0, \dots, n\}$,
 - $\Sigma = \{0, 1\}$,
 - $\Delta(0, 0) = \{0\}$, $\Delta(0, 1) = \{0, 1\}$, and $\Delta(i, a) = \{i + 1\}$ for all $0 < i < n$ and $a \in \Sigma$,
 - $S = \{0\}$, and
 - $F = \{n\}$.
- Describe the language accepted by N_n .
 - Show that there cannot exist a DFA $M_n = (Q_M, \Sigma, \delta_M, s_M, F_M)$ such that $L(M_n) = L(N_n)$ and $|Q_M| < 2^n$.
- (3) 4. Consider a language A over Σ . Prove that $\text{LT}(A) = \{y \mid xy \in A \text{ and } |x| = 2|y| \text{ for some } x \in \Sigma^*\}$ is regular whenever A is regular.