universität innsbruck

Automata and Logic	26W	LVA 703302 + 703303
Lecture 2		October 23, 2026

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. The (strict) deadline is 7 am on October 23.

Exercises

⟨2⟩ 1. Construct NFAs for the following sets. Try to take advantage of nondeterminism as much as possible.
   (a) The set of strings over $\Sigma = \{0, 1, \ldots, 9\}$ such that the final digit has appeared before.
   (b) The set of non-empty strings over $\Sigma = \{0, 1, \ldots, 9\}$ such that the final digit has *not* appeared before.

⟨2⟩ 2. Consider the following NFA$_\epsilon$ N:

		ϵ	a	b	c
$\to$	1	$\varnothing$	$\{1\}$	$\{2\}$	$\{1,3\}$
	2	$\{1\}$	$\{2\}$	$\{3\}$	$\varnothing$
	$3F$	$\{2,3\}$	$\{3\}$	$\varnothing$	$\{1\}$

   (a) Compute the ϵ-closure of each state.
   (b) List all strings in $L(N)$ of length two or less.
   (c) Transform N into an equivalent DFA without inaccessible states.

⟨3⟩ 3. In this exercise we show that NFAs are exponentially more succinct than DFAs, i.e., we prove that for any $n > 0$, there exists an NFA with $n + 1$ states such that any equivalent DFA has at least 2^n states. Let $n \geqslant 1$ and consider the NFA $N_n = (Q, \Sigma, \Delta, S, F)$ with
   - $Q = \{0, \ldots, n\}$,
   - $\Sigma = \{0, 1\}$,
   - $\Delta(0, 0) = \{0\}$, $\Delta(0, 1) = \{0, 1\}$, and $\Delta(i, a) = \{i + 1\}$ for all $0 < i < n$ and $a \in \Sigma$,
   - $S = \{0\}$, and
   - $F = \{n\}$.

   (a) Describe the language accepted by N_n.
   (b) Show that there cannot exist a DFA $M_n = (Q_M, \Sigma, \delta_M, s_M, F_M)$ such that $L(M_n) = L(N_n)$ and $|Q_M| < 2^n$.

⟨3⟩ 4. Consider a language A over Σ. Prove that $\mathrm{LT}(A) = \{\, y \mid xy \in A \text{ and } |x| = 2|y| \text{ for some } x \in \Sigma^* \,\}$ is regular whenever A is regular.